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Book Review: Nonequilibrium Phenomena I, The Boltzmann Equation

Nonequilibrium Phenomena I, The Boltzmann Equation. Edited by J. L. Lebowitz and E. W. Montroll. North-Holland Publishing Co., Amsterdam, 1982.

Readers of the *Journal of Statistical Physics* interested in the properties of the Boltzmann equation will find this an interesting and useful volume. The range of topics covered illustrates the strong and central role that the Boltzmann equation still plays in our understanding of nonequilibrium processes in gases.

The book begins with a short paper by Lanford on the derivation of the Boltzmann equation for particles interacting with short-range forces in the Boltzmann–Grad limit,[1] starting from the Liouville equation. Lanford's paper is a guide to the ideas used in the proof of the central theorem, namely, that one can justify the Boltzmann equation in this limit for times up to one fifth of the mean free time. While his theorem does not allow one to justify the Boltzmann equation for its usual range of applicability, it is nevertheless a very deep and very interesting study. The paper here allows one to get a clear idea of what goes into the proof and to understand the subtleties and difficulties involved in a rigorous justification of the Boltzmann equation from the basic principles of mechanics.

Given the Boltzmann equation as a starting point, one would also like to know whether or not it has solutions and what the properties of such solutions would be. The general question of existence proofs for the Boltzmann equation is discussed in the paper of Greenberg, Polewczak, and Zweifel. Here global existence theorems are surveyed starting with the classic work of Carleman in 1933 through the more recent work of Arkeryd, Ukai, and others. The discussion is in the form of theorems that

[1] The Boltzmann–Grad limit is the limit $n \to \infty$, $a \to 0$ such that na^2 remains finite. Here n is the average number density of the particles, and a is a characteristic size of a particle. This ensures that the mean free path of the particles remains finite while the reduced density na^3 approaches zero.

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have been proven with an indication of what the main ingredients in the proofs are.

The paper by Ernst surveys the activity that followed the Bobylev, Krook, Wu solution of a model nonlinear Boltzmann equation. His article describes the class of model one-dimensional Boltzmann equations that are amenable to exact solutions or otherwise suitable for numerical studies. He discusses the relevance of the particular Bobylev, Krook, Wu similarity solution for constructing the general solutions of such Boltzmann equations, and the describes one of the main general results from this work, namely, the nonuniform approach of an arbitrary initial distribution function to the solution of the linearized Boltzmann equation and eventually, to the Maxwellian. The last part of his paper addresses the application of these methods to polymerization and coagulation problems. This subject is one of considerable current interest and Ernst shows the rather striking connection between aspects of Boltzmann equation research and research in this area. Of particular interest is the relevance of exotic particle and energy nonconserving solutions of the Boltzmann equation to coagulation and polymerization physics.

Ernst's paper is followed by Cercignani's excellent survey of solutions of Boltzmann equation for boundary value problems of special interest in fluid and gas dynamics. His paper is a starting point for one to learn about mathematical techniques for solving the Boltzmann equation with physically interesting boundary conditions. While the relevant literature is much too extensive for a full discussion in a paper, Cercignani does an exemplary job of summarizing it and directing the reader to the appropriate source for detailed presentation.

Of course, one of the central themes of the Boltzmann equation is that it provides a kinetic basis for a derivation of the equations of fluid dynamics. The mathematical features of this derivation are reviewed in the paper of Caflisch. Here, too, one worries about the general properties of the Hilbert and Chapman–Enskog expansions of the solutions of the Boltzmann equations. Theorems on the convergence of such expansions and of the behavior of such solutions near boundaries are reviewed. In addition, Caflisch provides a nice but brief discussion of the mathematical questions arising in the application of Boltzmann and hydrodynamic equations to a study of shock waves.

The final paper in this volume, by Spohn, provides a nice counterpoint to the first, by Lanford. Spohn is concerned with the mathematical description of fluctuations in a dilute gas. The problem of interest here is the following: The distribution function appearing in the Boltzmann equation does not describe the actual μ-space density of any particular system, but instead it describes the mean value for a statistical ensemble of similar systems. In what sense this description is valid and under what limiting conditions is, of course, the subject of Lanford's paper. Now the μ-space density for any laboratory system will not be described by this function, but rather will deviate from it by a presumably small amount. Then one writes the μ-space density as the sum of two terms, one exact in some limit and one expressing fluctuations about the limiting value. The precise characterization of the fluctuations is described in Spohn's paper. Here one will find a careful discussion of the Boltzmann–Langevin equation and related topics including a number of interesting questions about fluctuations in non equilibrium steady states. As a relatively new area of investigation in Boltzmann equation research, the theory of fluctuations raises a set of questions not covered by the other papers and it will certainly stimulate many readers of the volume to think about this subject seriously.

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